SCORE-TIME and REAL-TIME

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Compiler's note: Due to space limitations, it was unfortunately not possible to reproduce these appendices in this volume. Interested readers are invited to contact the authors for further details.

Two Kinds of Scores

We are all familiar , as "computer musicians", with the task of translating notes into samples. A user normally inputs his notes in a "score", executes various "note transformation and reordering" routines, and finally executes his "orchestra". In MUSIC4-type programs, note transformation and reordering tasks are performed in PASS1 and PASS2, while the user's ORCHESTRA is called and executed during PASS3. All these PASSes affect "musical time". In this paper, we will concern ourselves only with the "note-transformation and reordering" tasks of PASS1 and PASS2, leaving aside the temporal considerations introduced by the user's "orchestral design".

We are thus concerned with two types of scores. The PASS1 score reflects time as specified by the user; the PASS2 score is the transformation of the PASS1 score into "clock" time. If the tempo is unchanging or changing in discrete steps, there is no problem in computing this transformation. To compute the duration of a beat, we simply compute the "period" of tempo.

Eq. 1 Dur <secs/beat>=60<seconds/min>/tempo<beats/min>

When the tempo is "changing continuously", we encounter complexities in performing our desired transformations. Even the concept of the "duration of a beat" is initially puzzling.

Graphing Music's Times

Let us assume that the "beat line" of a musical passage is continuous and that we may speak of "beat points" in a manner analogous to the way we speak of points on a line. There are thus an infinite number of beat points on the beat line. Further, between any two beat points there are an infinite number of beat points. There is a tempo associated with every beat point in a passage. Similarly, there is a "period" associated with every beat point. This period is simply the duration that one beat would have at the associated tempo. We will henceforth call this period the "clock factor" of a beat point.

Example 1 shows a graph of the linear increase of elapsed time vs beats when the tempo is a steady 60 beats per minute. If we were to graph this steady tempo vs beats, we would simply produce a straight line parallel to the "beat axis". Example 2 shows a graph of clock factor vs beats for 6 beats at a tempo of 60 beats per minute and 6 beats at a tempo of 90 beats per minute. Note a point of fundamental importance; the area under the clock factor curve equals elapsed time in seconds. Thus:

1. The clock factor is the derivative of, or instantaneous rate of change in, elapsed time vs beats.
2. Elapsed time is the integral of, or area under the curve formed by, clock factor vs beats.
3. A steady tempo is one in which each beat point has the same clock factor.
4. An acceleration occurs when each beat point of a passage is associated with a smaller clock factor (and larger tempo) than the preceding beat point.
5. A ritard occurs when each beat point of a passage is associated with a larger clock factor (and a smaller tempo).

The shape of tempo change and the shape of clock factor change will rarely be of the same type. Example 3 graphs a linear tempo change from 60 to 120 over 12 beats. Example 4 shows the hyperbolic clock factor curve associated with this tempo change. Example 5 graphs a linear, clock-factor accelerando from 1. to .5 over 12 beats; Example 6 shows the associated hyperbolic tempo curve.

It is easy to compute the "temporal area" associated with a linear clock factor. The "durational area" of Ex. 5 is obviously 9 seconds.

Equations 2 and 3 are used to compute a linear tempo and its associated duration at some beat point, bp. The durational area of Ex. 4 is 8.3177 seconds.

Eq. 2 $T(bp) = (K * bp) + C$

where T(bp) is the tempo at some beat point; and K and C are constants given T1 (the first tempo) and T2 (the ending tempo).

$T(0) = (K*0) + C$; thus $C = 60$.

$T(12) = 12K + 60$; thus $K = 5$.

Eq. 3 $DUR(bp) = (60./K) * \ln(T(bp)/T1)$

$DUR(12) = (60/5) * \ln(2) = 12 * .6931$;

$DUR(12) = 8.3177$.

The Equal Ratios Curve

Eq. 4 shows the equal ratios function applied to tempo.

EQ. 4 T(bp) = T1 * ((T2/T1)**(bp/B))

where B is the total number of beats in the passage.

The TVAL program of MUSIC4's PASS2 evaluates tempo change using the equal ratios curve. The tempo program we will describe later will be "biased towards" equal ratios. There are good reasons for this "equal ratios" preference in tempo relations. Perhaps the strongest is that we tend to interpret tempi related by powers of 2 in an analogous manner to the way we interpret the relationship of quarter notes, eighth notes, sixteenth notes, etc. The equal ratios curve also possesses the following attractive mathematical properties:

1. Its reciprocal is equal ratios. This means that if tempo is increasing by equal ratios, clock factor will be decreasing by equal ratios.
2. Its integral is also equal ratios. This means that the "real-time" durations of successive beats will be related by equal ratios.
3. It is symmetric under reflection. This means that a ritard from tempo 2x to 1x is exactly the same as the retrogression of an accelerando from 1x to 2x.

Example 7 shows an equal ratios accelerando from 60 to 120 over 12 beats. Example 8 graphs the changing clock factor associated with this accelerating tempo and thus shows us "temporal area". Eq. 5 is the expression for calculating the "equal ratios" duration at some beat point.

Eq. 5 DUR(bp)=(60/T1) * (B/ln(T1/T2)) * ((T1/T2)**(bp/B)-1)

The duration under the clock factor curve in Ex. 8 is thus 8.656 seconds, showing that equal ratios produces a duration longer than that produced by a linear tempo increase but shorter than that produced by a hyperbolic one.

Three Special Cases

1. Deriving Unknown Equal-Ratics Tempi

Suppose we know, with reference to some passage, its duration in seconds, its beginning or ending tempo (but not both), and the fact that it employs the equal ratios curve. Since we know that the clock factor at some bp is the derivative of elapsed time with respect to beats, it is possible to use numerical techniques to differentiate the missing clock factor and its associated tempo. These techniques are used in the tempo routine we will describe shortly.

2. Unequal Ratios

There are many situations to which the equal ratios tempo curve cannot be applied. Sometimes the passage is "overspecified" in the sense that one knows the beginning tempo, the ending tempo, and the total duration in seconds. If the duration does not agree with that produced by equal ratios, a different curve must be used. In other cases, another curve may be desired simply for aesthetic reasons.

It is not clear at present how many different types of curves need to be integrated to serve as the basis for tempo computation. It is possible to produce close approximations of most desired curves by "ordered distortions" of equal ratios curves. These distortions may be obtained by raising the equal-ratios, clock-factor values to odd powers (even powers produce singularities and other types of problems),"averaging" adjacent "power distortions", and "reflecting" the distorted curves so produced about a "linear tempo axis".. Curves whose slope changes sign can usually be treated in segments over which the clock-factor curve is monotonic. These techniques are incorporated in the tempo subroutine we will describe shortly.

3. The Inverse Equal-Ratios Curve

An equal ratios tempo change gets "faster faster" and "slower more slowly". This may cause problems. If it does, the "inverse" equal ratios curve may prove to be helpful. The inverse equal ratios curve increases by the same amount the retrograde of the equal ratios curve decreases.

Eq. 6 T(bp) = T1 +(T2 - (T1 * (T2/T1)**((B-bp)/B)))

The duration of an inverse equal ratios curve may be obtained by using Eq. 7.

Eq. 7 D = DLT - (DER - DLT)

where
D is the inverse equal ratios duration,
DLT is the duration associated with a linear tempo change, and
DER is the duration of an equal ratios tempo change.

An A-Subroutine for Tempo Manipulation

The TVAL program and MUSIC4's PASS2 contain two basic temporal problems. First, since tempo is applied to events after they have been sorted into a presumed temporal order, it is very difficult to produce independent simultaneous tempi. A second and more serious difficulty is connected to the "quantization" of tempo change. TVAL computes tempo change discretely, not continuously. The number of discrete segments used equals the number of PASS3 "time-segments". This means that passages will take up differing amounts of clock-time depending on the number and distribution of notes in the passage. TVAL thus does not have a unique solution to the question posed by several of our earlier examples, "Given a starting tempo, an ending tempo, and a shape of tempo change, what time is it at beat 12?"

We will incorporate our tempo transformations in a FORTRAN-language A-Subroutine which calls several other FUNCTIONS and SUBROUTINES, also in FORTRAN. The A-Subroutine is the standard method of PASS1 data modification in MUSIC4. The main advantage of PASS1 tempo processing is that it allows the user to operate on "the notes" in their "entered order" before they have been sorted into a presumed temporal order. This means that simultaneous "different tempi", ritards, accelerandos, etc. are easy and straight-forward. In order to keep all parts coordinated, our routine must evaluate time continuously and be able to integrate the function shapes it needs.

APPENDIX I of this article contains a printout of an extensively commented version of our tempo routines. Several special features of the programs are discussed there in more detail than we can go into in this talk. APPENDIX II shows many extensively-documented, PASS1-PASS2 examples of the uses of this program. For the present, we will give a brief overview of the way one normally uses the program. This discussion must assume that the user is familiar with data handling in MUSIC4's PASS1 since a discussion of this subject would far exceed the time we are allocated.

We define the beginning of a PASS1 time-segment to be the beginning beat point of the segment; the end of the time-segment is the beginning beat point of the next segment. A particular tempo is associated with the beginning beat point of each time segment. A duration in seconds is also associated, either implicitly or explicitly, with each time segment. The direction of tempo change may change between PASS1 time-segments, but cannot change within time segments. One "note" will often participate in many different time-segments. Each PASS1 time segment is characterized by:

[<beat point in beats> <tempo in beats/min> <dur in secs>].

We will call a field which contains these three pieces of information a "T-field". T-fields are divisions of our PASS1 tempo card. Special features of the A-subroutine may be activated by certain "unusual" values being placed in a T-field.

1. If the dur-in-secs parameter is 0, we will assume equal ratios is the desired shape of tempo change.

2. If a positive dur-in-secs value is present, we will derive a tempo change curve which results in an elapsed time equal to the given duration, if such a curve is possible. If the curve is impossible, an error message will be generated for the user and equal ratios will be used.

3. If a tempo is negative, we will take this as a clue to supply a "missing equal-ratios tempo". The absolute value of the "negative tempo" will be taken as the dur-in-secs value of the passage.

In addition to these special cases, the program handles the following:

1. If two successive tempo fields are the same, we recognize a steady tempo over the time segment.

2. If two successive beat point fields are the same, we recognize an instantaneous tempo change.

3. The "normal" case is for beat points and tempi to be changing while durations remain 0.

The PASS1 tempo card and the notes on which it operates should be entered using the "N-card" mechanism. This is described in detail in APPENDIX I. The A-routine for tempo manipulation is called A2. Various kinds of housekeeping data may be entered on the A-card itself. These options are described in APPENDIX I; here we will content ourselves with the normal case in which the only data the A-Routine recives on its "calling card" is the number of the routine, 2.

Example 9 (which is also Section 8 of APPENDIX II) shows a typical run of the A2 subroutine. The PASS1 printout shows the input score and the T-card asking for an equal-ratios accelerando from 60 to 120 over 12 beats. The PASS2 printout shows the actual durations produced by the operation of A2. Note that the time at the beginning of beat 12 is 8.656 seconds. Eq. 4 may be used to verify the correctness of the other durations and starting times.

A Passage from Elliott Carter's Double Concerto

Consider Ex. 10, a passage from Elliott Carter's Double Concerto. The piano is accelerating from 140 to 210 over 9 beats while the remaining parts hold a steady tempo of 140 for 8 beats. The interim tempi of the accelerando are almost, but not quite, equal ratios. (A tempo of 160 instead of 159 would give equal ratios.) The duration of the passage is determined by the steady tempo of the non-piano parts and is 3.4286 seconds. An equal ratios accelerando produces far too short a duration for the piano, 3.171 seconds. We may try to divide our curve into three separate equal ratios curves and aim them at Carter's interim tempi. This produces 3.182 seconds, a duration which is still almost a beat off in the new tempo.

A more reasonable total time for the passage is given by simply playing successive 3 beat units in the "steady" tempo indicated and changing "instantly" to the next tempo at the indicated interim tempo location. This "upper limit" interpretation gives a duration of 3.401 seconds and is very close to the necessary duration, though still slightly too short.

Several other solutions to this passage suggest themselves:

1. Tempo change might be introduced into the "steady parts" so that their duration could be made to agree with any desired duration in the piano.

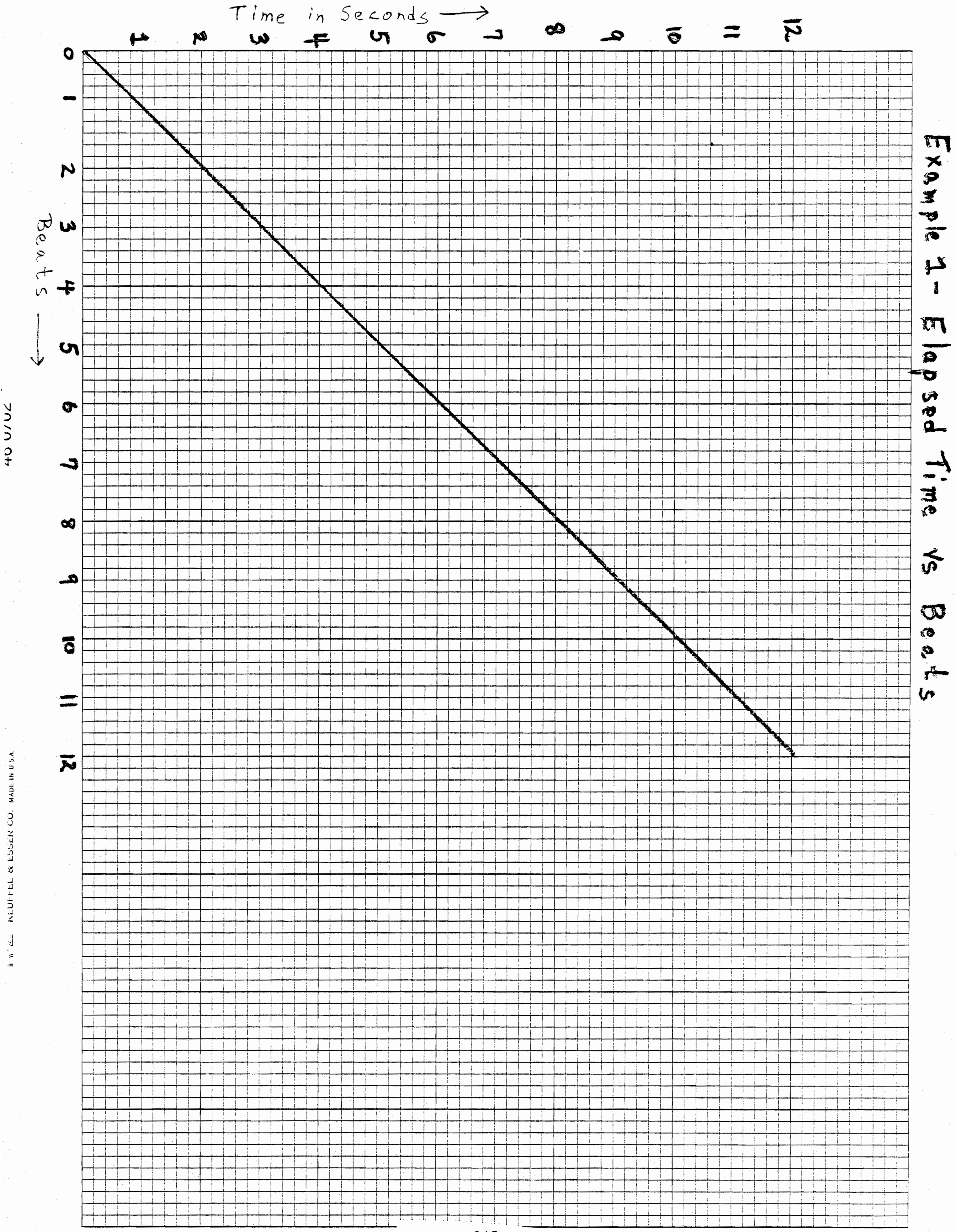
2. The beginning or ending tempo of the piano could be changed so that equal ratios tempo change could be used.

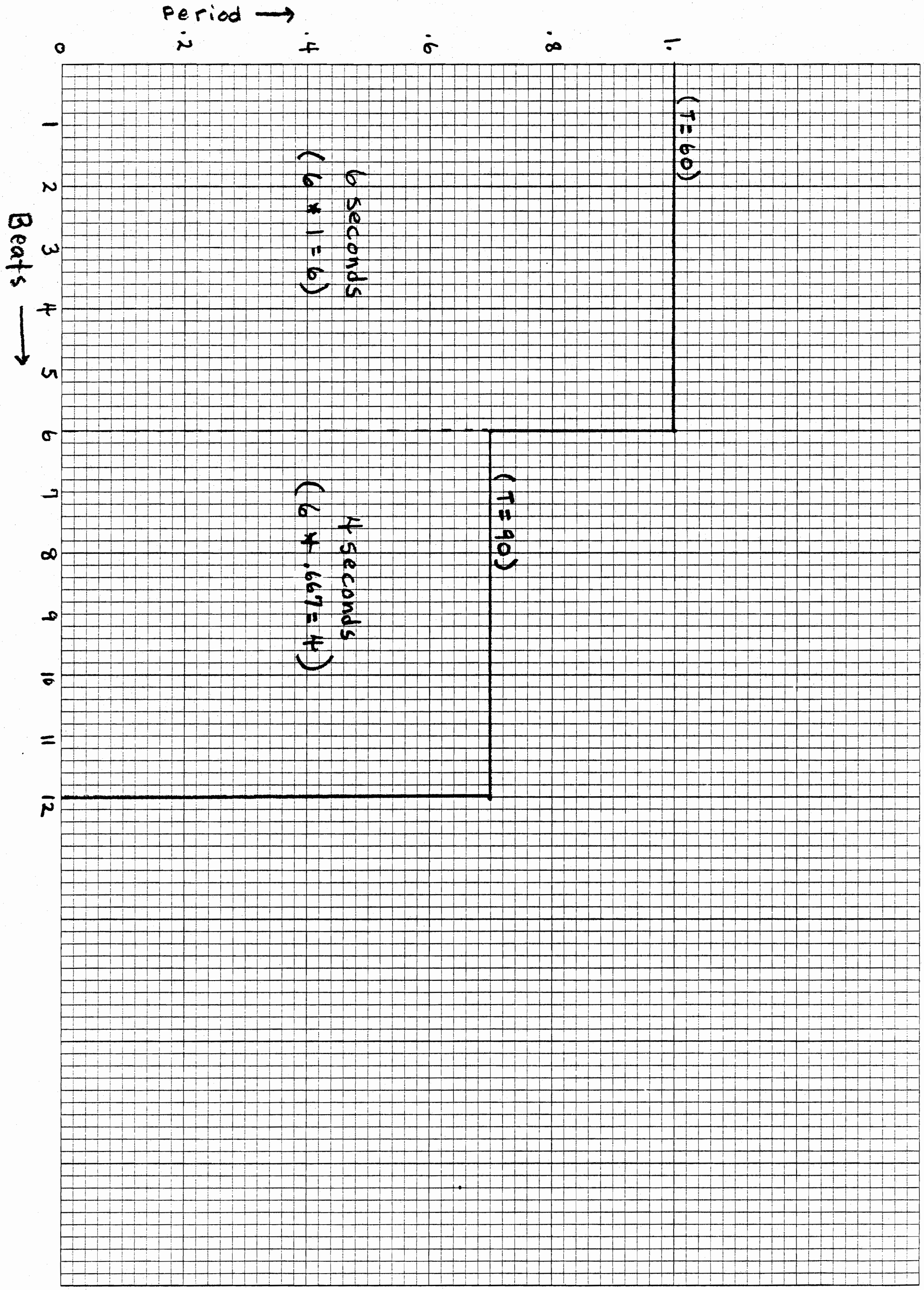
3. Different time-segments could be considered so that equal-ratios could still be used to produce the desired duration.

4. A "distorted" version of equal ratios could be used to produce a tempo curve that will give the desired duration. APPENDIX III contains several related PASS1/PASS2 realizations of this passage using this option in the call to A2.

Conclusions

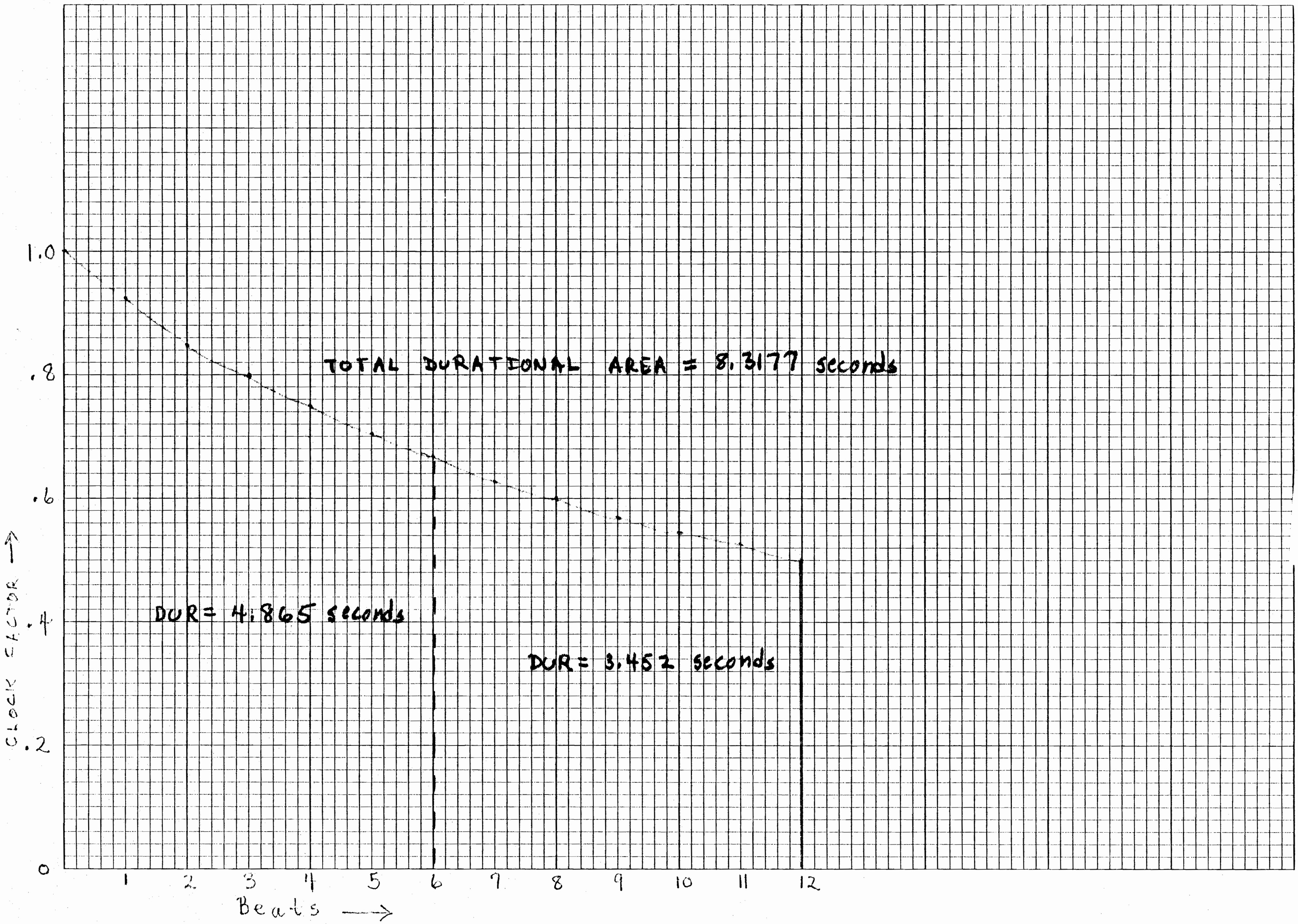
The paper you have just heard represents a synoptic and selective view of material treated more fully in the authors' "A Question of Time". Several articles on the general topic of time and music have appeared recently in journals of music theory. We would like to believe that all this activity signals a willingness on the part of theorists and composers to try once again to grapple with this fundamentally important and puzzling relationship. Thus, in conclusion, we offer the suggestion that a panel be devoted to the topic of "Time and Computer Music" at the next meeting of this organization.

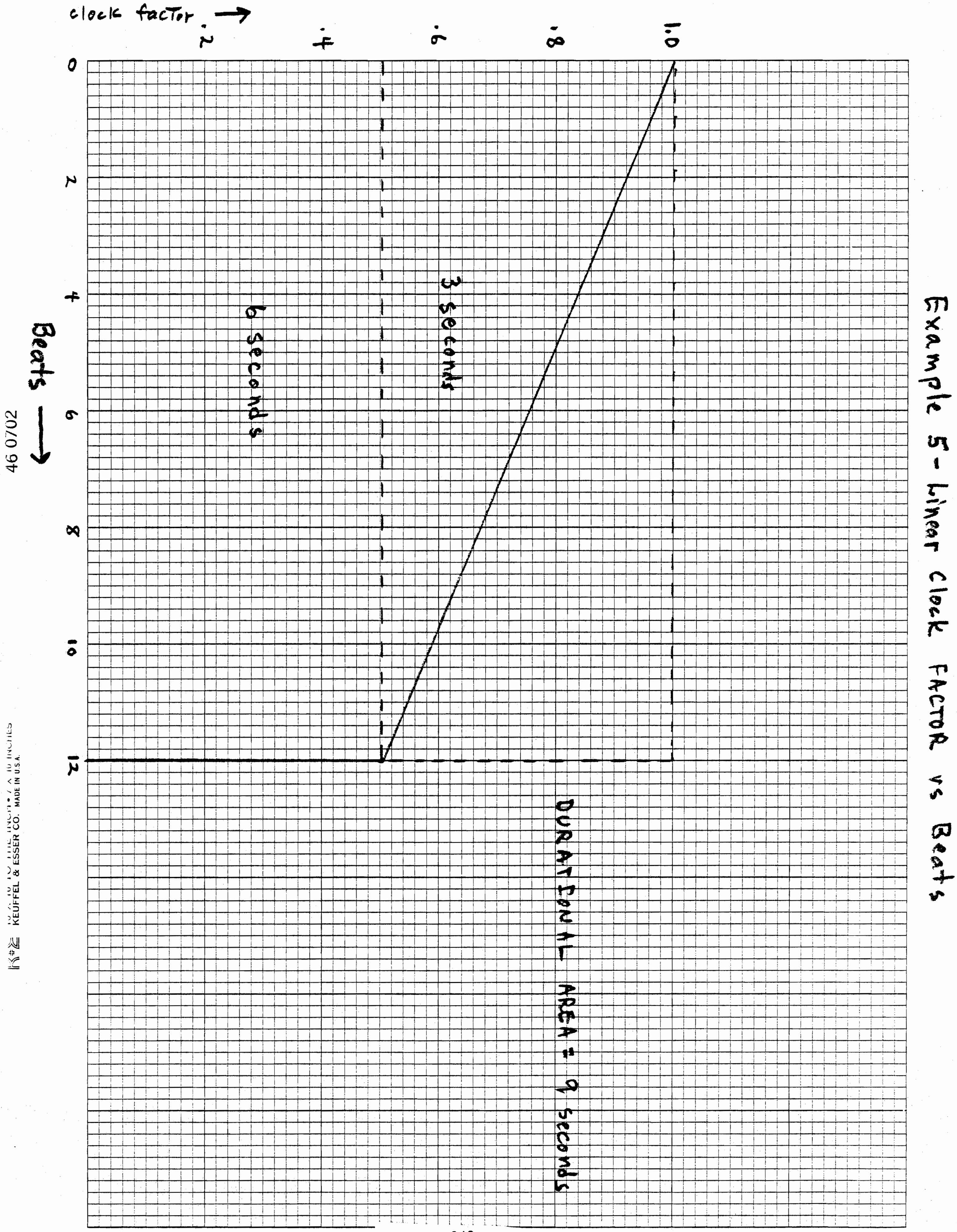

Time in Seconds
1 2 3 4 5 6 7 8 9 10 11 12
Beats
0 1 2 3 4 5 6 7 8 9 10 11 12
Example 1 - Elapsed Time vs Beats
KEUFFEL & ESSER CO. MADE IN U.S.A.

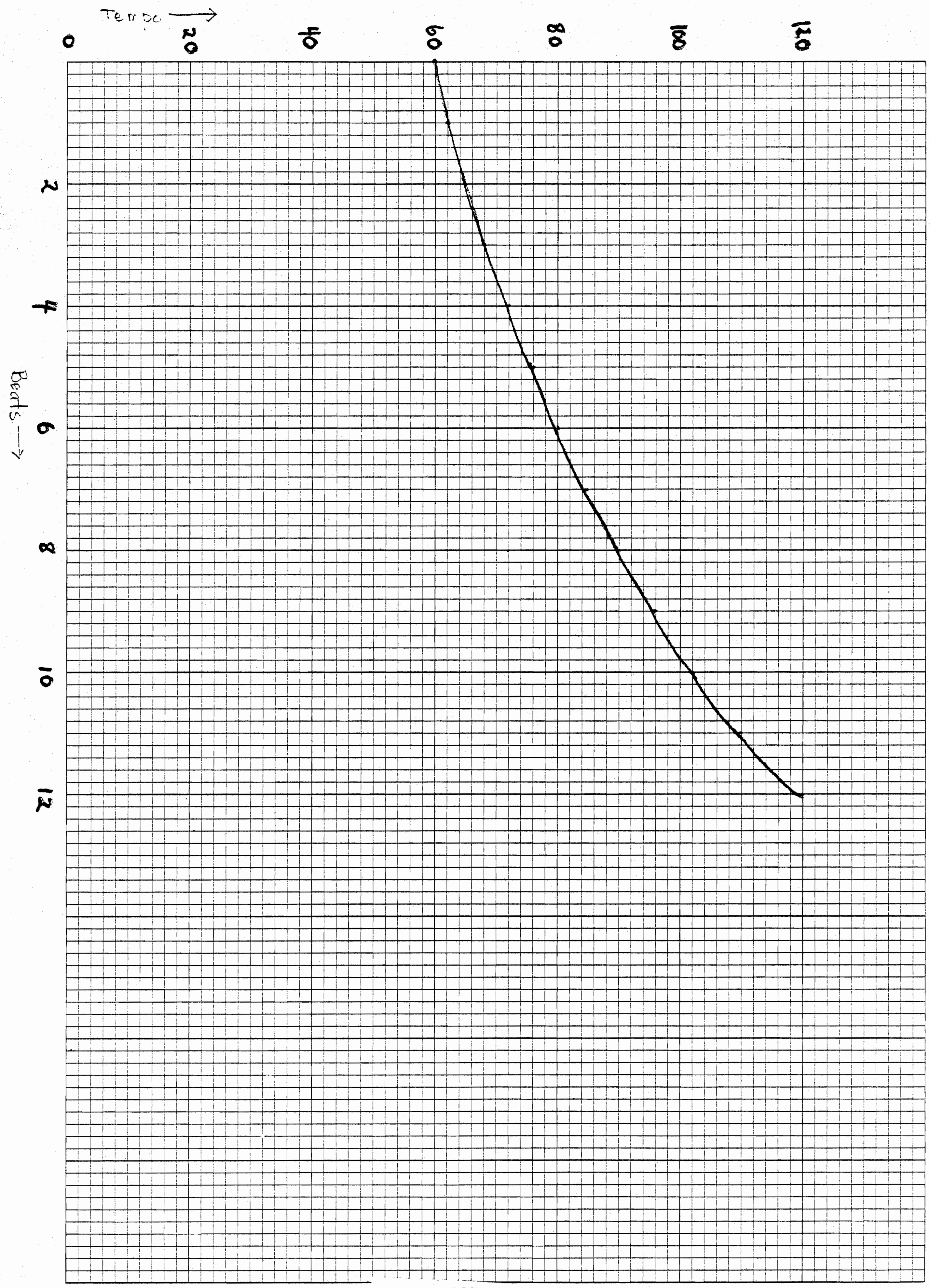

Period →
0
.2
.4
.6
.8
1.
Beats →
1
2
3
4
5
6
7
8
9
10
11
12
(T = 60)
6 seconds
(6 * 1 = 6)
(T = 90)
4 seconds
(6 * .667 = 4)


Example 3 - Linear Tempo vs Beats
120
100
80
60
40
20
Tempo →
0
2
4
6
8
10
12
Beats →

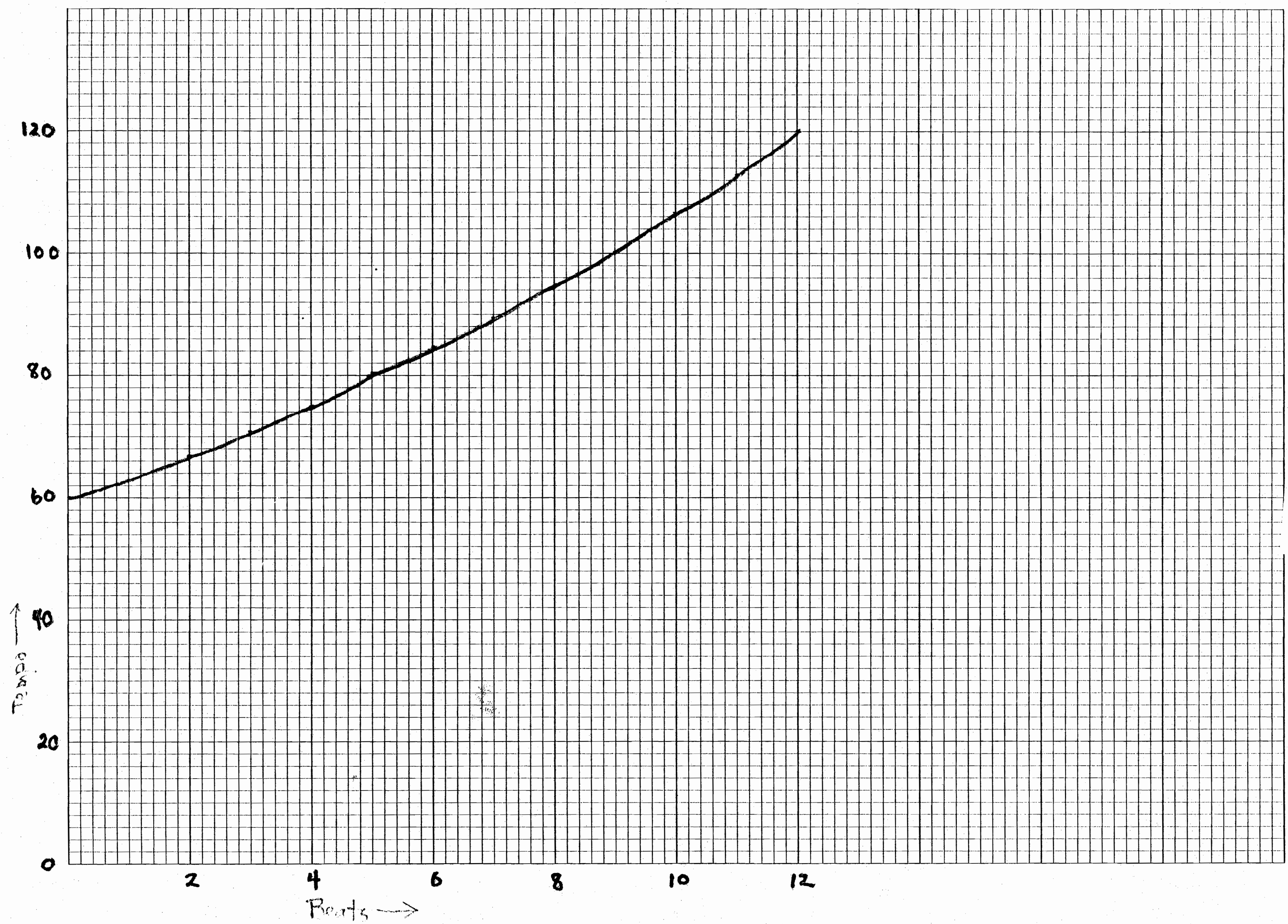
Example 1 Hyperbolic Clock factor vs Beats

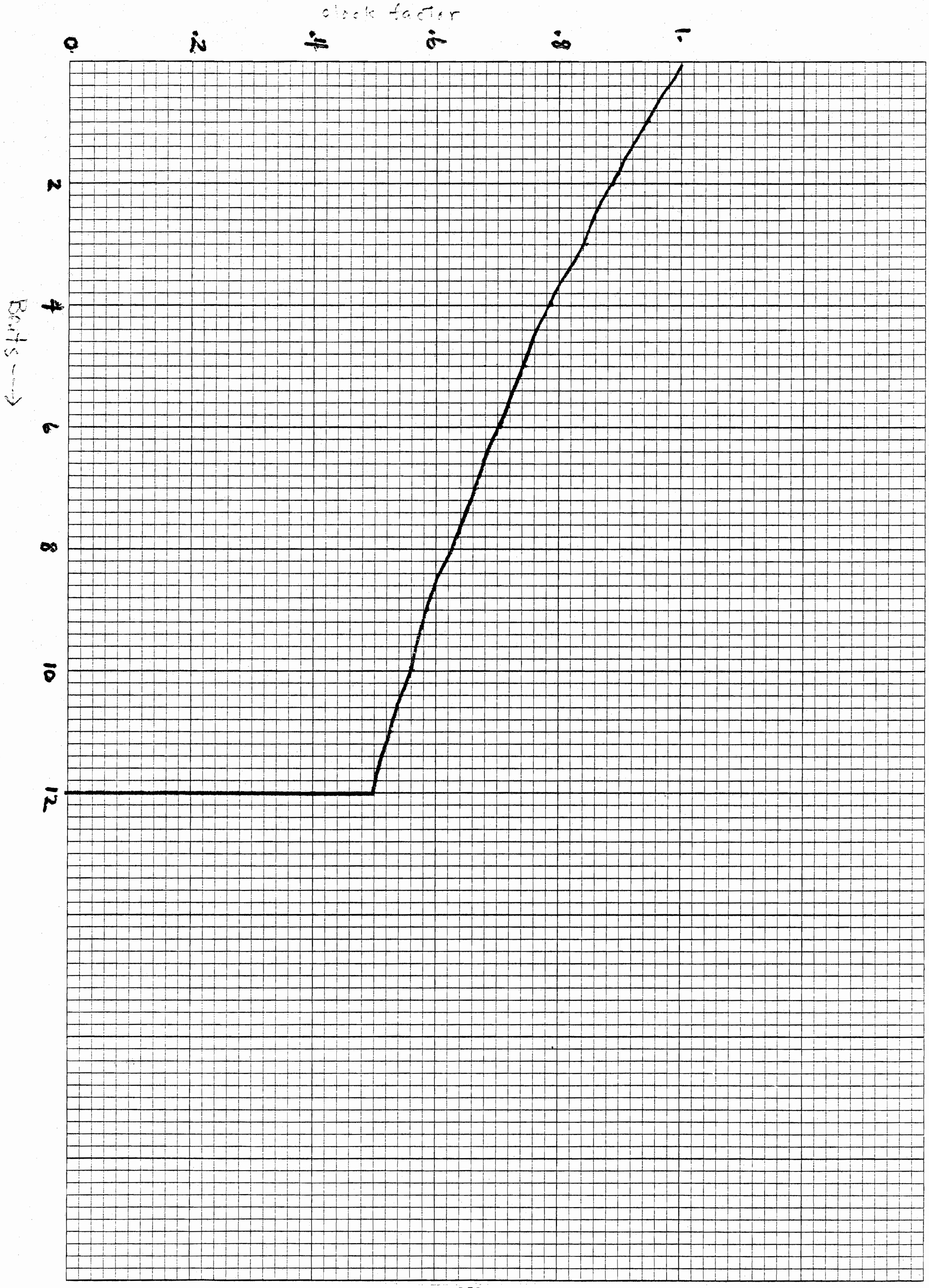



Example 5 - Linear Clock FACTOR vs Beats
clock factor
.2
.4
.6
.8
1.0
Beats
0
2
4
6
8
10
12
6 seconds
3 seconds
DURATIONAL AREA = 9 seconds
46 0702
KEUFFEL & ESSER CO. MADE IN U.S.A.


Tempo →
0
20
40
60
80
100
120
Beats →
2
4
6
8
10
12

Example 17 - Equal Ratios Tempo vs Beats




clock factor
0
.2
.4
.6
.8
1
Bits
2
4
6
8
10
12

Example 9 (Section 8 of the APPENDIX II) Rogers & Rockstroh

```
CTION NUMBER 8****
           C      EQUAL-RATIOS ACCELERANDO FROM 60 TO 120 OVER 12 BEATS
           N 1    1.
           T990010.    60.    0.     12.    120.   0.
           N 2    0.
           N 3    -2.
           I 40020.    1.     500.   7.00   .01    .05    1.     -.09   1.1    2.5
           I 40031.    1.     500.   7.01   .01    .05    1.     -.09   1.1    2.5
           I 40042.    1.     500.   7.02   .01    .05    1.     -.09   1.1    2.5
           I 40053.    1.     500.   7.03   .01    .05    1.     -.09   1.1    2.5
           I 40064.    1.     500.   7.04   .01    .05    1.     -.09   1.1    2.5
           I 40075.    1.     500.   7.05   .01    .05    1.     -.09   1.1    2.5
           I 40086.    1.     500.   7.06   .01    .05    1.     -.09   1.1    2.5
           I 40097.    1.     500.   7.07   .01    .05    1.     -.09   1.1    2.5
           I 40108.    1.     500.   7.08   .01    .05    1.     -.09   1.1    2.5
           I 40119.    1.     500.   7.09   .01    .05    1.     -.09   1.1    2.5
           I 401210.   1.     500.   7.10   .01    .05    1.     -.09   1.1    2.5
           I 401311.   1.     500.   7.11   .01    .05    1.     -.09   1.1    2.5
           I 401412.   1.     500.   8.00   .01    .05    1.     -.09   1.1    2.5
           N 4    0.
           A 2
           A10    -5.    -6.
           B 1    0.     6.     6.     24.    24.    6.
           S
```

```
OSECTION NO.    8
****AN ACCURATE EQUAL RATIOS ACCELERANDO****
   I    37.         0.000       0.972     500.000        7.000
   I    37.         0.972       0.917     500.000        7.010
   I    37.         1.889       0.866     500.000        7.020
   I    37.         2.754       0.817     500.000        7.030
   I    37.         3.572       0.771     500.000        7.040
   I    37.         4.343       0.728     500.000        7.050
   I    37.         5.071       0.687     500.000        7.060
   I    37.         5.758       0.649     500.000        7.070
   I    37.         6.406       0.612     500.000        7.080
   I    37.         7.018       0.578     500.000        7.090
   I    37.         7.596       0.545     500.000        7.100
   I    37.         8.141       0.515     500.000        7.110
   I    37.         8.656       0.500     500.000        8.000
```

Example 10

Adding up durations vs performing an integration seems to indicate tempo changes at bar x bar increments, not continuously!

♩=140; CF = 60/140 = .42857

♩=159; CF = .37736

♩=183; CF = .32787

6139-168